Board of Education
Palo Alto Unified School District

I am writing to give my impressions of the program Everyday Math, which is being considered for adoption in the Palo Alto district. I have had three kids go through the district (last one graduated in 2005), and continue to watch the development of the mathematics program with interest. Everyday Math certainly has strengths, but my feeling is that it misses the mark in certain important aspects, for example in the area of algorithms for basic arithmetic operations. I will first briefly describe my view of the reasons for teaching these algorithms to students.

One item of controversy in many discussions of mathematics education is the relative valuation of algorithmics/computation on the one hand and various notions of conceptual or higher level thinking on the other hand. I believe this is a false dichotomy, since algorithmic thinking and conceptual thinking support each other in very direct ways. Fluency with algorithmics and computation provide the familiarity with concrete consequences of conceptual thinking which solidifies otherwise abstract notions, and conceptual understanding provides both motivation for computation as well as the ability to detect obvious errors in computation. This fluency with computation plays much the same role as fluency in the study of languages, or in the development of musical skills. Fluency in language frees the mind to consider the subtle ideas represented by various turns of phrase, and in music permits one to interpret pieces of music rather than simply reproducing them mechanically. On this basis, I favor that students be taught "optimized versions" of the algorithms for computation, i.e. the standard methods which are understood to be the simplest and quickest to implement in manual computation. The importance of simplicity should be clear, but the value of speed is also crucial. If the computations are carried out in a much too laborious or time-consuming fashion, their value in supporting conceptual thinking will be diminished. Think of a method for teaching piano in which one is taught the logical structure of chords and scales, but never is given sufficient practice to play fluidly and quickly. To actually play the piano requires instant recognition of chords, and if that recognition process is too slow, producing good music becomes impossible. Similarly, if one were taught the definitions of words and grammar, if one does not have a mechanism for decoding the printed page which is relatively instantaneous, the process of reading becomes too laborious and will be avoided by students. Such fluency in the mathematical domain is critical to success in algebra and later in calculus, where a lack of such fluency will greatly hamper the student's ability to problem solve and therefore ultimately his/her ability to understand the underlying ideas.

My view is that Everyday Math introduces idiosyncratic methods for performing addition, multiplication, and long division, and that this is done because in the view of the authors these methods are easier to explain conceptually than the standard, optimized, ways. They also believe that these methods are somehow easier on the students. For example, the Partial Quotients Division algorithm is described as a "low-stress" algorithm. The first point is a good argument for introducing these methods as interesting examples which can clarify theory, but it is **not** a good argument for teaching these algorithms as the algorithms of choice for performing all computations. For addition, the book introduces a "partial sums" algorithm for computing sums, which has the effect of lengthening computations in most cases. The authors describe the standard method as being suitable for struggling students. In fact, the standard algorithm is simply a quick and effective short hand version of the partial sums algorithm, which is valuable for all students. The differences between the Partial Products Multiplication method for multiplication introduced in Everyday Math and the standard algorithm are much more pronounced. It breaks up many of the steps in the standard algorithm and in general increases the number of required additions significantly. The argument in favor of this method is that it makes clearer the role of the distributive property in performing multiplication. This is useful to point out, but one should not require students to "hold one hand behind their back" by avoiding the standard shorthand which speeds up these calculations. The Partial Quotients Division algorithm is similarly awkward.

An additional point is that the structure of the standard algorithms provides a great opportunity to exercise conceptual understanding. Each shorthand step illustrates a valuable application of a fundamental property of numbers, and a thorough analysis of each such algorithm from this point of view would do a great deal to solidify students' understanding of the fundamental properties of arithmetic.

I have only discussed my observations about algorithmic thinking, but I certainly have misgivings about other aspects of the program, such as early use of calculators and the handling of fractions, but have not had chance to look at them in detail.

In summary, the authors of Everyday Math feel that it is worthwhile to trade computational fluency and speed for having algorithms in which the underlying properties of arithmetic stand out in the clearest possible way. I do not regard this as a necessary or desirable trade-off. The connections of the theory with the algorithms are important to demonstrate, but need to be done only when the algorithms are introduced. One does not need to saddle students with less effective methods in order to remind them of this connection every time they perform a calculation.

Sincerely yours,

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